Theoretical Reasoning and Development of The Mathematical Model of Teacher's Reflective Activity

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**Abstract**

The aim of this study is theoretical reasoning and development of models of teacher's reflective activity as a feedback system. Two principles were the basis for the implementation of the model. They are consistency (observation of the educational process as a multi-level, functional system) and activity (system of activity). There were used mathematical apparatus of the theory of probability, in particular the method of fuzzy set theory in the processing of linguistic values. The study defines the content of the model of teacher's reflective activity. It is proved that the reflective activity of the teacher is a feedback system. The consideration of reflective activity as a feedback system opens up new possibilities for its abstract description. The description of a pedagogical reflection in the form of a mathematical model, representing it as a multi-dimensional vector space of individual qualities has the same purpose. The presented mathematical model allows using the mathematical apparatus of the theory of probability for measuring the level of reflection.

**Keywords:** Reasoning, Development, Reflective Activity, pedagogy.
Introduction

The topicality of the problem of reflective education in Russian and foreign pedagogy is associated with the priority of the individual as the main imperative of the modern educational system. The strategy of innovative development of the Russian Federation comprises the requirements to the training of competitive and qualified professionals: the development of learning skills on their own, focus on professional development, critical thinking. These requirements and their continuous expansion have a significant impact on the functioning of the professional education system. The modern approach to the organization of educational process at pedagogical high school is to focus on reflective activities of its members and the creation of conditions aimed at the interested attitude to reflective behavior, the acquisition of professional reflective knowledge by future specialists. The improving of competitiveness and efficiency of education depend on the professional level of teachers' staff. Professionalism of teachers is one of the key conditions of the student's personality development, his successful socialization and it ensures the formation of a qualitatively new system of education.

The realization of the given objectives is possible only with the appropriate teachers' training.

Main Part

The Strategy of Russian Federation's Innovative Development defines the basic requirements to the training of qualified, competitive specialists: readiness to continuous professional growth, formation and development of skills of independent acquisition of knowledge, critical thinking. The constant increase in these requirements affects the organization of training in the system of professional education. The specifics of modern approach to the organization of educational process in pedagogical high school, oriented on teacher and students' reflective activity, should be to create conditions that ensure acquiring of professional reflective knowledge, interested attitude to a reflective activity, stimulating future professional's reflective behavior.


Differences in opinion are manifested in the experience of the use of certain teaching methods and in emphasis placement [Games and simulations / Keis Fh.C. & Selleart Jo. He.-S-F, 1991. 283 p.]. It should be noted the presence of different approaches contributes to more comprehensive and deeper disclosure of the essence of the reflection.

The presence of two similar positions is also traced in the interpretation of reflective processes that are typical for foreign and Russian pedagogy. They are: a) a reflective analysis of consciousness that leads to the interpretation of the object values and their constructing; b) reflection as the understanding of the meaning of interpersonal communication. In this regard, there are the following reflective processes: self-evaluation and evaluation of the others; understanding of the others and self-understanding; the interpretation of the others and self-interpretation.

Based on the foregoing, we can conclude that a foreign science has obtaining the experience of reflective practice of education since the second half of the 1980s, while Russian pedagogy has suspended the development of techniques and methods of reflection. That's why domestic and foreign sciences face different problems now. Thus, for the Russian pedagogy of higher school the most urgent is the development of new technologies.

We define a pedagogical reflection as a kind of teachers' professional activity. This definition is based on a psychological theory of activity developed by Ananiev B.G., Vygotsky L.S., Davydov V.V., Zaporozhets A.V., Leontiev A.N., Rubinstein S.L.

The theory represents the psychological structure of activities which main elements are necessity, motive, purpose, conditions for achieving the goals and concepts of activity, action and operation correlated with them.

The developed model of teacher's reflective activity is as follows:

**Goal setting**: the formation of the creative style of professional activity, the improvement of scientific and practical activities.

**Functions**: analytic (focused on the analysis of the performed activities), control (aimed at identifying the discrepancies of new results or new situation to the current style); constructive (aimed at forming a new principles of action in the individual's mind).

**Content**: Motivational-target unit – goals, motives, needs, stimulating reflective activity of the teacher.

Procedural and maintaining block – psychological and pedagogical knowledge, reflective skills and experience.

Emotional and volitional unit – personal and professional qualities important for the reflective activity.

Control and evaluation unit – analytical, control and constructive function of the teacher's evaluating activity.

**Types**: personal, interpersonal, object-functional and methodological reflection.

**Levels**: high, medium, low.

**Results**: professional competence, improving the effectiveness of pedagogical activity, the growth of pedagogical skill level and creative approach to the activity [Stetsenko I.A. Pedagogical reflection theory and the development of technology in the future teachers. Taganrog, 2003. 228 p.].
The model of the teacher's reflective activity is based on two principles: the consistency (examination of the educational process as a multilevel functional system) and activity (as the system of activities) [Morozova O.P. Teaching Dictionary Directory. Barnaul, 2000. 256 p.].

In this article we consider the teacher's reflective activity as a system with a feedback (see. Figure 1).

So, the teacher solves certain pedagogical situation in accordance with existing knowledge, abilities and skills, or creates it for a student with to identify the effectiveness of the learning process. Reflective teacher analyzes the obtained result in accordance with the existing teaching experience and corrects it, in case of dissatisfaction with the result. The link between pedagogical situation and pedagogical reflection is carried out with the help of images, structure and organizational degree of which depends on the level of teacher's consciousness. While reflexing there are changes in the content of his personal experience. When considering this model, we can see signs of a system with a feedback.

Using the analogy with the systems in systems engineering, we can say that the reflective activity determines the feedback factor, i.e., the completeness and quality of the output action analysis. By output action is understood the result of solving educational problems [Gorokhov V.G. Methodological analysis of systems engineering. M.: Radio and Communications, 1982. 134 p.]. Spreading the results well-known from the theory of systems on the represented structure of the reflective activity use, it can be assumed that this structure with a closed-loop feedback will provide the right solution of more challenges than the system without feedback (absence of reflective activity). Moreover, the range of tasks will be greater, the better teacher's knowledge, abilities and skills will be adapted into a solution of a specific task and therefore the higher «feedback gain». In their turn knowledge, abilities and skills will be set the more accurately to the solution of the problem, the better the reflective part of the teacher's consciousness reflects the problem essence. This structure will be non-linear, since by virtue of the experience and abilities each teacher can solve the educational problems, limited in complexity.
Each of the units of represented structure (see Figure 1) has no defined mathematical description; therefore, we can’t talk about the deterministic description of the whole structure of reflective activities model. Thus the probabilistic model is proposed. This model allows determining quite distinct patterns and characteristics while studying such a complex object.

Basically the concept of probability has «idealized» asymptotic nature. By definition, the probabilities are the values of a certain real function defined on the event class, which represent the results of tests (observations). The probabilities are introduced by certain axioms based on the asymptotic continuation of the concept of relative frequency of an event in an infinite number of tests (experiments, observations).

It should be noted that the strict mathematical sense of probability coincides with the relative frequency (which has a simple «physical», «ordinary» sense) only with infinitely number of tests - observation, thus the probability gains its «idealized» sense. The use of fuzzy set theory methods, e.g., while processing linguistic variables, can be an effort to avoid this drawback when describing the observations. But we should note that at the moment they have a fairly narrow application.

Next, we describe the proposed mathematical model of reflection estimation based on the following axiomatic basis:

1. There is an n-dimensional Euclidean space of reflection denoted as $\mathbb{R}_n$. $\mathbb{R}_n$ space is a set of points representing by the vector ends.

2. In this space reflection, which j-th object possesses is displayed by vector $x_j = (x_1, x_2, \ldots x_n)$, defined as a set of numbers - the levels of individual qualities (properties) of the object. The set of these qualities is the basis in which the reflection is determined [Mihailychev E.A., Mehantsev B.E. Mathematical methods in pedagogical research. M.: Higher School, 2008. 215 p.].

In practice, instead of deterministic coordinate values, we obtain their estimate values - random variables, thus, a reflection is a random vector in $\mathbb{R}_n$ space (see Figure 2).

In Figure 2 the end of the vector with a certain probability may be in a particular area of space. This probability can be determined by knowing the distribution of $x_1, \ldots x_n$ values. The end of the vector is not a certain point, but a cloud, which variable density expresses the probability of the vector end to be in a given volume element. Herewith the reflection is an n-dimensional random value, and its level is determined by the norm (generalized length) of a random vector. A distribution function $F(x_1, \ldots x_n)$ is a universal characteristics, suitable to describe both discrete and continuous values. This function can be interpreted as the probability of hitting of a random point inside the infinite lower left quadrant the with the vertex $(x_1, \ldots x_n)$, t. e. $F(x_1, \ldots x_n) = P(X_1 < x_1, \ldots X_n < x_n)$ [ ].

![Figure 2. The random vector in $\mathbb{R}_n$ space](image-url)
The probability density function \( p_n(x_1, \ldots, x_n) \) is convenient to describe the distribution of system of continuous random values. Distribution law of value system is determined by the distribution of each value in the system and the relationship between them. Mathematically it can be shown that under the conditions of normality and independence of the laws of the vector coordinate distributions (i.e., individual characteristics), the distribution law of its module (i.e., level of reflection) \( r_{\text{ref}} \) is as follows:

\[
R_{\text{ref}}(x) = 2x^n - 1 \frac{x^2}{2\sigma^2} \left( \begin{array}{c} n \end{array} \right) e^{x^2/2} G \left( \frac{n}{2} \right)
\]

where \( x \) – level of reflection,
\( \sigma \) – standard deviation of estimation of individual property,
\( G \left( \frac{n}{2} \right) \) – gamma function.

For independent coordinates, e.g. in two-dimensional case, the degree of dependence of random values \( x_1 \) and \( x_2 \) is characterized by the conditional distribution law of one of the random value, obtained under the condition that the other random value gained a certain significance. \( p_2(x_1, x_2) = p_1(x)p_1(y|x) \) for two random, where \( p_1(y|x) \) is the conditional density of probability. If the values of \( x_1 \) and \( x_2 \) are independent, then \( p_2(x_1, x_2) = p_1(x_1, x_2) \).

Within the viewing mathematical model, the dependence of random values displays the existing links of individual qualities, measured to determine the reflection. Quantitatively these links are defined by the correlation matrix \( ||K|| \).

\[
||K|| = \begin{bmatrix}
K_{x_1 x_1} & K_{x_1 x_2} & \ldots & K_{x_1 x_n} \\
K_{x_2 x_1} & K_{x_2 x_2} & \ldots & K_{x_2 x_n} \\
\ldots & \ldots & \ldots & \ldots \\
K_{x_n x_1} & K_{x_n x_2} & \ldots & K_{x_n x_n}
\end{bmatrix}
\]

Each of the matrix elements (2), e.g. \( K_{x_1 x_3} \), is the centerpiece of a mixed moment of second order of the random values \( x_1 \) and \( x_3 \) of individual quality levels, determines the degree of their linear dependence and is defined by the integral:

\[
K_{x_1 x_2} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x_1 - m_{x_1})(x_2 - m_{x_2})p_2(x_1, x_2)dx_1dx_2
\]

According to the theory of statistical decisions, acceptance of the hypothesis about increasing of the level of reflection should be based on the results of the analysis of the likelihood ratio:

\[
\Lambda [ x_n ] = p_n(x_1, \ldots, x_n | H_1) / p_n(x_1, \ldots, x_n | H_0),
\]

where \( p_n(x_1, \ldots, x_n | H_1) \) and \( p_n(x_1, \ldots, x_n | H_0) \) – n-dimensional density of probability distribution of random vectors, with the truth of the hypothesis about increasing of the level of reflection and absence of increasing (zero - hypothesis) respectively.

If the result of the analysis shows that the likelihood ratio exceeds a certain \( C \) value, the hypothesis \( H_1 (H=H_1) \) will be true, otherwise the hypothesis \( H_0 (H=H_0) \) will be true. The confirmed hypothesis means the partition of the whole area of possible values \( x_1, \ldots, x_n \) into two subareas \( G_1 \) and \( G_0 \).
When the obtained combination $x_1, \ldots, x_n$ quality levels (end of a random vector) gets in area $G_1$, decision $H = H_1$ is made, while getting into area $G_0$ leads to the decision $H = H_0$. Different criteria may be used to calculate the threshold $C$, but we choose only one. It is Neyman-Pearson criterion, according to which the best is a rule that maximizes the probability of making correct decisions $H = H_1$, $P_{D1}$, with a given probability of false alarm $P_{F0}$, i.e. making $H = H_0$, whereas $H = H_0$ is the true.

Criteria study goes far beyond the scope of this paper, so we only note that the calculation of the threshold $C$ is associated with the use of knowledge about the form of the distribution law of a multidimensional vector of reflection, which in turn depends on the laws of distribution of random values of their coordinates [Fukunaga K. Introduction to statistical recognition theory. M.: Nauka, 1979. 368 p.].

The mathematical model reflects a definition adopted in the abstract form of certain complex dependence of reflection on the individual qualities of the subject. Reflection as a multidimensional value characterizes its relation with both separately taken quality and the interrelation of these qualities. It means that the position of a point in a multidimensional space is defined by the individual qualities that are its projections on the respective axes, and the relationships between these qualities, which are expressed by angles between the axes. The proposed mathematical model can serve as a basis for further development in order to identify practical ways of its implementation. The quality of the model corresponds to the model adopted in psychology. In psychological studies A.V. Karpova it is experimentally defined that while creating conditions for reflection intensifying the success of the subject's solution of a variety of problems increases significantly [Karpov A.V. Reflexivity as a mental property and methods of its diagnostics // Psychological magazine. 2003. Vol 24. №5. P. 45-47].

**Overview**

Reflection on the practice is the basis for the creation of improved practices, and the latter, in turn, encourages further reflection and the subsequent correction of the practice. The practice becomes a source of the teacher's professional growth only to the extent that it is subject of a structured analysis. Unreflected practice is useless and does not lead to the development, but to teacher's professional stagnation over time. Thus, pedagogical reflection is a process of sequential actions from difficulty (doubt) to its discussion with yourself and finding a way out of it.

However, reflection is not a feedback itself, just like an ordinary mirror, which reflects the person's appearance, is not a feedback itself, but means, method or mechanism by which this feedback can be obtained. Reflection as a feedback mechanism in human life is not only a result (the reflection in the mirror), but also the process associated with the internal transformations – reflection and reconsideration of activities, introduction of corrective actions.

**Conclusion**

Thus, consideration of reflective activity as a feedback system opens up new possibilities for its abstract description. The description of a pedagogical reflection in the form of a mathematical model, representing it as a multi-dimensional vector space of individual qualities has the same purpose. The presented mathematical model allows using the mathematical apparatus of the theory of probability for measuring the level of reflection.

**Conflict of Interest**

The authors confirm that the submitted data does not contain the conflict of interest.
References