Applying Integrated FAHP-DEA Model for Managerial pure Efficiency Evaluation in Banking System

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Abstract

Nowadays, managerial performance evaluation in comparison with other managers is an important issue for decision makers and stakeholders in order to evaluate the objectives' achievement. They can also increase their productivity by having the result of evaluation in mind. This paper aims to propose a model based on DEA (Data Envelopment Analysis) and FAHP (Fuzzy Analytic Hierarchical Process). This integrated (FAHP-DEA) model is presented as a fundamental efficiency measure in the evaluation of bank branches' performance. The proposed model is applied for the efficiency evaluation of 38 Iranian I&M bank's branches for calendar year 2013. The FAHP technique is utilized to determine inherent weights of efficiency criteria taking the subjective judgments of experts into account. Then DEA method is used for ranking bank's branches considering the inherent weights of criteria regarding confidence interval. The LINGO 14 was used to traditional DEA and FAHP-DEA integrated efficiencies analyses. The result revealed that there are significant differences between CRS (Constant Return to Scale) and VRS (Variable Return to Scale) based on traditional DEA and FAHP-DEA integrated model.

Keyword: Fuzzy AHP, DEA, FAHP-DEA and Pure Efficiency Analysis

1. Introduction

At the present time, efficient performance of banking system plays dominant role in any country’s growth and development. Banks are those governmental and private financial institutions which gather customers savings and allocate them as credit to the firms, corporations, plants and so on. By this means, they participate in producing goods and services. Therefore, powerful managerial performance in banking system is related to both efficient gathering of savings of customers and efficiently allocating them to the economic activities. Thus, responsible managers, officials and stakeholders look for the best techniques to evaluate the performance of banking system’s managers. This role is carried out for any bank branches by their supervision and evaluation department annually.

According to Supervision and Evaluation Department ranking of bank branches of Iran is as following:

- Ranking based on savings.
- Ranking based on allocated credits
- Ranking based on bad and doubtful debts

Considering weakness of such rankings, applying efficient models that use effective criteria has been rising in importance. This study aims to evaluate Iranian I&M bank’s branches across the country by combining MCDM(multi-criteria decision making) models. Bank's branches (managerial) performance can be measured using some basic ratios as indicators, coming from branches performance evaluation statements. These statements are issued by Supervision and Evaluation Department. These ratios provide valuable foundation for complete efficiency analysis of managers of Bank's branches by decision makers, too [1].

Mansouri and salehi used traditional DEA method for bank's branches grading in Iranian I&M bank branches [2] and a comprehensive paper by Berger and Humphrey[3] reviewed the literature concerning the efficiency appraising of financial institutions, including bank branches, using non-parametric (DEA and variations) and parametric frontier analysis. Lovell and Pastor[4] considered setting targets for bank branches; Camanho and Dyson[3] evaluated Portuguese bank's branches using data envelopment analysis and discussed technical efficiency analysis. Golany and Storbeck [5] examined operational efficiencies in bank branches. Oral and Yolalan[6] examined 20 branches of Turkish commercial bank where DEA was used to reallocate resources among branches. Building on the previous work, Sherman and Gold [7], Sherman and Ladino[8] reported that applying DEA results at restructuring process of 36 US branches of a bank, had led to actual annual savings of over $6 million. Zenios et al.[9] studied the Bank of Cyprus where the bank adopted their model and findings to establish policy guidelines and provide operational support for productivity improvements. Then, Athanassopoulos and Giokas [10] examined 47 branches of the Commercial Banks of Greece and DEA results were used to implement the proposed changes in the banks’ performance measurement system. Another study has been done by Cook et al.[11]. When they applied DEA to evaluate the large Canadian Banks branches and the bank accepted their new performance rating system based on DEA. The papers all dealt with non-discretionary variables in real situations. However unrestricted DEA can yield quite unrealistic results from a managerial point of view and there are situations where additional information is available that allows the analyst to impose conditions on the components of the multiplier vectors. Recently Chansarn[12] conducted a survey that entitled the relationship between efficiencies for commercial banks in Thailand. For this purpose he considered the efficiencies of 13 commercial banks in Thailand between 2003 and 2006, using data envelopment analysis. In this study, there were one input of personnel
number, and two outputs of incomes (total revenue and net revenue) in DEA model. He used a CRS to measure efficiencies. The results of the study indicated that the efficiency for the banks with functional approaches, though growing gradually, is very high. In fact, the average of their efficiency has been more than 90%, yearly which seems to be unreal. Fotio et al.[13] presented a paper entitled "estimating and analyzing the cost efficiency for Greek cooperative banks: an application of two stage data envelopment analysis". In this paper, they estimated the efficiency for 16 Greek cooperative banks from 2000 until 2004. First, they estimated cost, specialized, technical, and management efficiency, using DEA method. Then, they recognized internal and external factors affecting these kinds of efficiency, using probit regression method. The results of their study revealed that the banks were inefficient 17.7% in average.

Here, to assess Iranian I&M bank's branches efficiency, a comprehensive and intelligent approach is proposed. This approach is introduced as FAHP-DEA integrated model which uses both fuzzy logic and AHP technique to improve DEA capability. Using proposed model, the efficiency evaluation for bank's branches can be effectively conducted. For this purpose, the present paper is organized as follows: section 2 introduces functional ratios used in this study. In section 3, a summary of the proposed methodology will be presented. In section 4, model experimentation and results is described. The sensitivity analysis is given in section 5. Finally, conclusion and acknowledgment parts of paper can be found in section 6 and 7.

2. Functional ratios

Basically, functional ratios are key elements showing the primary measures refereed as DMU’s(Decision making unit’s) performance indices by experts and professionals. They let specialist users analyze DMU’s situation and attain meaningful information for their decision-making[14]. Effective functional ratios are useful instruments that can be used for efficiency assessment of DMUs. They also help decision makers to determine proper strategies and know, how their enterprise is able to accomplish its verified objectives[15]. Recognizing effective efficiency indicators is a vital section in efficiency analyses. Thus, to attain primary managerial performance indicators for DEA model, they should be divided into three clusters of desirable, undesirable and moderator variables[16]. So, following categories are suggested:

2.1. Desirable indicators

These indicators show DMU’s desired performance in given period of time. Decision makers strongly try to increase the value of desired variable to enhance their performance. These are known as output variables in DEA model. There are three commonly used desired indicators in baking system as follow:

- **The customers growth** which is primary indicators that guarantees bank’s branch success. This indicator implies managerial customer-orientation performance in a given period.
  
  \[
  \text{Customer growth} = \frac{S_t - S_{t-1}}{S_{t-1}}
  \]
  
  Where \(S_t\) is the current customers numbers and \(S_{t-1}\) is the last period’s customers numbers.

- **The revenue growth** which shows possible changes in total revenue of bank’s branches in a specified period of time. This ratio demonstrates managerial financial performance.
Revenue growth = \( \frac{R_t - R_{t-1}}{R_{t-1}} \)  
(2)

In which, \( R_t \) is the current revenues and \( R_{t-1} \) is the last period's revenues.

- **The net revenue growth** which is the ratio of net revenue growth to the last period's net revenues. This ratio directly illustrates managerial financial capability and competencies.

Net revenue growth = \( \frac{N_t - N_{t-1}}{N_{t-1}} \)  
(3)

Where \( N_t \) is the current net revenues and \( N_{t-1} \) is the last period's net revenues.

- **The credit growth** which is the amount of credit growth to the last period's allocated credit.

This ratio directly shows managerial performance in credit allocation.

Credit growth = \( \frac{C_t - C_{t-1}}{C_{t-1}} \)  
(4)

Where \( C_t \) is the current allocated credits and \( C_{t-1} \) is the last period's allocated credits.

### 2.1. Undesirable ratios

These indicators imply DMU's operational and non-operational costs' management in a given period of time. Decision makers, forcefully, try to reduce the value of these variables without any changes in the desired indicators to enhance their performance. These kind of variables considered as input variables in DEA model. There are two commonly used undesirable indicators in baking system as follow:

- **The operational cost growth** which is changes on operational cost to the last period's operational cost. This ratio shows managerial performance in operational cost managing.

\[ \text{Operational cost growth} = \frac{O_t - O_{t-1}}{O_{t-1}} \]  
(5)

In which, \( O_t \) is operational costs and \( O_{t-1} \) is the last period's operational costs.

- **The non-operational cost growth** which is changes on non-operational cost to the last period's non-operational cost. This ratio shows managerial performance in non-operational cost managing.

\[ \text{Non-operational cost growth} = \frac{NO_t - NO_{t-1}}{NO_{t-1}} \]  
(6)

In which, \( NO_t \) is non-operational costs and \( NO_{t-1} \) is the last period's non-operational costs.

### 2.2. Moderator variables

In addition to desirable and undesirable variables that are well-known in management performance evaluation areas, there are some important variables that indirectly influence managerial performance in a given situation. These variables can be referred to as moderator variables. There is an important ratio in the banking system which has taken a great deal of considerations, in can be found below.

The **population density** which is an important ratio that affects any manager performance. This ratio shows a number of competing financial institutions to relative population number within a radius of one kilometer which is attained through GIS(Geographical Information System). Great number of this ratio makes difficult the performance improvement for any bank’s branch. Thus, it considered as input variable in DEA Model.
Thus, input and output variables of DEA model can be summarized as the following table:

Table 1
Input and output variables of DEA model with their measures, abbreviations and descriptions

<table>
<thead>
<tr>
<th>Variable and its measures</th>
<th>Abbreviation</th>
<th>Input/output</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customers growth (%)</td>
<td>$Y_{1j}$</td>
<td>Output</td>
<td>Customers growth for DMU $j$</td>
</tr>
<tr>
<td>Revenue growth (%)</td>
<td>$Y_{2j}$</td>
<td>Output</td>
<td>Revenue growth for DMU $j$</td>
</tr>
<tr>
<td>Net revenue growth (%)</td>
<td>$Y_{3j}$</td>
<td>Output</td>
<td>Net revenue growth for DMU $j$</td>
</tr>
<tr>
<td>Credit growth (%)</td>
<td>$Y_{4j}$</td>
<td>Output</td>
<td>Credit growth for DMU $j$</td>
</tr>
<tr>
<td>Operational cost growth (%)</td>
<td>$X_{1j}$</td>
<td>Input</td>
<td>Operational cost growth for DMU $j$</td>
</tr>
<tr>
<td>Non-operational cost growth (%)</td>
<td>$X_{2j}$</td>
<td>Input</td>
<td>Non-operational cost growth for DMU $j$</td>
</tr>
<tr>
<td>Population density (%)</td>
<td>$X_{3j}$</td>
<td>Input</td>
<td>Population density growth for DMU $j$</td>
</tr>
</tbody>
</table>

3. Proposed methodology

In the current study, a strong methodology for managerial efficiency growth evaluation is proposed. For this purpose, at the first step, pivotal growth ratios were detected. These ratios for a calendar year, shows managerial efficiency improvement, exclusively. Fuzzy AHP was used to determine inherent weights of input and output variables. The weighing results of Fuzzy AHP were applied into the CCR (Charnes, Cooper and Rodes) and BCC (Banker, Charnes and cooper) version of DEA model to prevent unrestricted variables fluctuations, resulting in miss efficiency evaluation. Consequently, the final weights of input and output variables weights were approached to their inherent values and all DMUs' efficiencies also tended to reality. Necessary steps of proposed approach are demonstrated in Fig. 1.
3.1. Fuzzy logic

Fuzzy set is a class of numbers with grades of membership function which is between zero and one [17]. Fuzzy logic based on fuzzy set theory is proposed to deal with reasoning which is approximate rather than precise. It permits the model to incorporate many verbal issues - experts’ advice- in expansion of parameters estimates[18]. It enables decision maker to handle uncertainty. There exist many types of fuzzy numbers. Triangle fuzzy numbers (TFN) is the most popular kind of fuzzy number. The TFN is usually demonstrated with $\tilde{A} = (l, m, u)$ where $l$, $m$ and $u$ show the minimum, the most likely and the maximum possible value of a fuzzy number, respectively and
In which the membership function is a continuous mapping from \( R \) to the interval \([0,1]\). Some of the main mathematical operations for two TFNs numbers have been expressed as follows:

\[
\begin{align*}
\mu_{\bar{A}}(x) &= \begin{cases} 
\frac{x-l}{m-l} & l \leq x \leq m \\
\frac{m-x}{u-m} & m \leq x \leq u \\
0 & x < l \text{ or } x > u 
\end{cases} 
\end{align*}
\]  

(8)

In which \( x \in (-\infty, +\infty) \) and \( \mu_{\bar{A}}(x) \) is a membership function which is a continuous mapping from \( R \) to the interval \([0,1]\). Some of the main mathematical operations for two TFNs numbers have been expressed as follows:

\[
\begin{align*}
\bar{A} + \bar{B} &= (l_1 + l_2, m_1 + m_2, u_1 + u_2) \\
\bar{A} - \bar{B} &= (l_1 - l_2, m_1 - m_2, u_1 - u_2) \\
\bar{A} \times \bar{B} &= (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2) \\
\bar{A}/\bar{B} &= (l_1/u_2, m_1/m_2, u_1/l_2) \\
k\bar{A} &= (kl, km, ku) \quad k > 0, \quad k \in R
\end{align*}
\]  

(9) (10) (11) (12) (13)

3.2. Fuzzy AHP

AHP is a decision making technique that decomposes the hierarchical problem to facilitate solving of complex MCDM(Multi-Criteria Decision Making) problems. AHP has been increasingly used to solve various MCDM problems. However, in many times uncertain information and vagueness of human feelings in the real world, makes AHP’s application inefficient. To manage such ambiguity and vagueness, Zadeh[17] introduced fuzzy sets theory. Thus integration of fuzzy concepts with AHP will be more beneficial and effective than classical AHP in solving the real world problems. There are variety of methods for fuzzification of AHP. Laarhoven and Pedrycz[19] fuzzified AHP for the first time. They used TFN to carry out pair wise comparisons. Buckley[20] applied trapezoidal fuzzy numbers to handle pair-wise comparisons. Chang [21] introduced a new method for AHP fuzzification; he utilized TFNs for pair wise comparisons scale and used extent analysis technique for synthetic extent values of pair wise comparing. Kahraman et al.[22] suggested subjective and objective fuzzy AHP method. In the current paper, TFNs were applied to fuzzify AHP to facilitate the decision making process [23]. The membership function of a TFN is shown in Eq.(8). After deciding on AHP fuzzyfication method, three steps are necessary.

Step 1: Determining criteria in hierarchical form

Similar to classical AHP, all criteria which seems to be affected by bank's branches managers performance should be determined. Since the main objective of current study focuses on management efficiency evaluation, the basic managerial efficiency criteria as ratio were detected. These important ratios have been illustrated in section 2.

Step 2: Collecting experts' ideas using TFNs and fuzzy pair wise comparison matrices

For this purpose, we must compare criteria as pair wise. Fuzzy AHP comparison matrix is shown in Eq.(14).The fuzzy AHP method uses pair-wise comparisons regarding fuzzy scale and linguistic expression of relative Importance between two criteria and values according to table 2.
In which, $a_{ij}^k = (1,1,1)$: $\forall i=j$; $a_{ij}^k = \frac{1}{a_{ij}}$: $\forall i\neq j$.

And $\tilde{A}^k$ is the fuzzy pair wise comparisons matrix of expert $k$, $\tilde{a}_{ijk}$ is a fuzzy comparison between criterion $i$ and criterion $j$ of expert $k$ which defines through TFN as:

$$\tilde{a}_{ijk} = (t_{ij}^k, m_{ij}^k, u_{ij}^k)$$

The linguistic scale for fuzzy pair wise comparisons is demonstrated in Table 2 based on Buckley [20] definition.

Table 2
Fuzzy Scale and Linguistic Expression of Relative Importance Between Two Criteria and Values of Strategies

<table>
<thead>
<tr>
<th>Intensity of fuzzy scale</th>
<th>Definition of linguistic variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equally important; very low</td>
</tr>
<tr>
<td>3</td>
<td>Weakly important; low</td>
</tr>
<tr>
<td>5</td>
<td>Essentially important; fair</td>
</tr>
<tr>
<td>7</td>
<td>Very strongly important; high</td>
</tr>
<tr>
<td>9</td>
<td>Absolutely important; very high</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate values between two adjacent judgments</td>
</tr>
</tbody>
</table>

Step 3: Defuzzyfication of any experts fuzzy pair wise matrices

There are many ways for defuzzyfication of fuzzy number. In the present paper, Liou and Wang’s[24] approach is applied to defuzzyfy, fuzzy matrix $\tilde{A}^k$ into the crisp matrix $G_{a,\mu}$:

$$G_{a,\mu} = \left[ g_{a,\mu}(\tilde{a}_{ij}) \right]$$

$$g_{a,\mu}(\tilde{a}_{ij}) = \left[ \mu \cdot f_{a}(l_{ij}) + (1-\mu) \cdot f_{a}(u_{ij}) \right]$$

$0 \leq \alpha, \mu \leq 1, \ i < j$ (17)

$$g_{a,\mu}(\tilde{a}_{ij}) = \frac{1}{g_{a,\mu}(\tilde{a}_{ij})}$$

$0 \leq \alpha, \mu \leq 1, \ i > j$ (18)

$$g_{a,\mu}(\tilde{a}_{ij}) = 1$$

$i = j$ (19)

$f_{a}(l_{ij}) = (m_{ij} - l_{ij}). \alpha + l_{ij}$ is the left-hand $\alpha$-cut value for a given $\tilde{a}_{ij}$ and

$$f_{a}(u_{ij}) = u_{ij} - (u_{ij} - m_{ij}). \alpha$$

is the right-hand $\alpha$-cut value for given $\tilde{a}_{ij}$.

Where $\alpha$ shows the range of uncertainty and reflects stable and unstable conditions. The greater the value for $\alpha$, the lower the degree of uncertainties is.
The $\mu$ indicates the degree of pessimism of a decision for matrix $\bar{A}^k$. The larger the value of $\mu$, the higher degree of pessimism or lower degree of optimism is. Thus, the defuzzyfied pair wise comparison matrix for any expert is shown as Eq. (20).

$$
\begin{bmatrix}
C_1 & C_2 & \ldots & C_n \\
C_1 & 1 & g_{\alpha,\mu}(\bar{A}_{12}) & \ldots & g_{\alpha,\mu}(\bar{A}_{1n})^k \\
C_2 & 1/g_{\alpha,\mu}(\bar{A}_{12}) & 1 & \ldots & g_{\alpha,\mu}(\bar{A}_{2n}) \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
C_n & 1/g_{\alpha,\mu}(\bar{A}_{1n}) & 1/g_{\alpha,\mu}(\bar{A}_{2n}) & \ldots & 1
\end{bmatrix}
$$

(20)

### Step 4: computing priority weights of criteria from any expert's viewpoint

In this step, similar to traditional AHP, priority weights of criteria will be computed from any expert's point of view. The priority weights of criteria from the point of view of expert $k$ can be shown as Eq. (21)

$$W^k = (w^k_1, w^k_2, \ldots, w^k_n)$$

Where vector $W^k$ shows expert $k$'s priority weights with respect to all criteria, $w^k_i$ is priority weight of criteria $i$ from expert $k$'s point of view.

### Step 5: computing mean, standard deviation and standard error of weights

Considering $W^k$ of all experts, mean, standard deviation and standard error of any criteria priority weights can be defined as Eqs. (22) to (24)

$$\bar{w}_i = \sqrt[k]{\prod_{k=1}^{n} w^k_i}$$

(22)

$$S_{w_i} = \sqrt{\frac{\sum_{k=1}^{n}(w^k_i - \bar{w}_i)^2}{n-1}}$$

(23)

$$S_{\bar{w}_i} = \frac{S_{w_i}}{\sqrt{k}}$$

(24)

Where $\bar{w}_i$ is sample mean value of criteria $i$, $S_{w_i}$ shows standard deviation of criteria $i$, $S_{\bar{w}_i}$ demonstrates standard error of criteria $i$ and $k$ shows the number of experts.

### Step 6: computing pair wise matrix consistency rate (C.R)

The initiator of AHP has presented a well-defined mathematical approach for testing conformity of pair wise comparisons matrix; To attain consistency rate (C.R.), eigenvalue ($\lambda_{\text{max}}^k$) of any expert’s pair wise comparison matrix should be computed. It can be computed by Eq. (25).

$$\text{Det}(g_{\alpha,\mu}(\bar{A}^k) - \lambda_{\text{max}}^k) = 0$$

(25)

$\lambda_{\text{max}}$ helps, to find consistency index (C.I.) and consistency rate (C.R.) using Eqs. (26) and (27).

$$C. I = \frac{\lambda_{\text{max}}^k-n}{n-1}$$

(26)

$$C. R = \frac{C. I}{R.I}$$

(27)

In Eq. (27) random index rate (R.I.) depends on square pair wise matrix order

In accordance with Saaty[25] Rule, C.R < 0.1 shows acceptable range of accuracy; otherwise matrix must be completed again.

3.3. Data envelopment Analysis (DEA)

DEA models are classified with respect to the types of envelopment surface, the efficiency measurement and the orientation (input and output). There are two basic types of envelopment surfaces in DEA: Charnes et al.[26] introduced the constant returns-to-scale (CRS) and Banker et al.[27] introduced the variable returns-to-scale (VRS) model. DEA models are also classified as radial input oriented, radial output oriented or integrated (both inputs and outputs are optimized) based on the direction of the projection at inefficient unit onto the frontier. DEA is a framework well suited for performance analysis and offers many advantages over previous methods such as performance ratios and regression analysis[12]. Largely the result of multi-disciplinary research during the last two decades in economics, engineering and management revealed that DEA is best described as an effective way of visualizing and analyzing performance data [28]. DEA is particularly effective in handling complex processes where DMUs utilize multiple inputs to produce multiple outputs. Technically, it represents a set of non-parametric and linear programming techniques which is used to construct empirical production frontiers and to evaluate the relative efficiency of units[27].

Clearly the efficiency of objective decision making unit (DMUo) which produces s services or products and uses m resources, can be defined as follows by DEA model:

\[
\begin{align*}
\text{Max} Z_0 &= \sum_{r=1}^{s} u_r y_{r0} \\
\text{subject to} & \sum_{i=1}^{m} v_i x_{i0} = 1 \\
\end{align*}
\]

Where \( u_r \) is the inherent value of output \( r \), \( y_{r0} \) shows the amount of output \( r \) for objective DMU, \( v_i \) is the inherent value of input \( i \), \( x_{i0} \) depicts the amount of input \( i \) for objective DMU.

Clearly, the larger value of fraction entails higher efficiency and vice versa but determination of inherent values for some inputs and output in many cases is impossible. Charns et al. [26] introduced a mathematical model which can evaluate DMU's efficiency without determination of inherent value of input and output variables. They proposed the model as CCR(Charns, Cooper and Rodes) version for decision making, summarized as follows:

\[
\begin{align*}
\text{Max} Z_0 &= \sum_{r=1}^{s} u_r y_{r0} \\
\text{subject to} & \sum_{i=1}^{m} v_i x_{i0} = 1 \\
\end{align*}
\]
In this model objective function shows objective DMU's efficiency and constraints depicts that no DMUs' efficiency can exceed from 1. The $u_r$ and $v_i$ can get any positive value. Also $y_{rij}$ and $x_{ij}$ are model parameters, extracted from DMUs' data centers and $u_r$ and $v_i$ must be determined by model in order to maximize DMU's efficiency. The formulation of CCR version assumes that the relationship between inputs and outputs follows the constant returns to scale assumption. For instance, if inputs get twice as much, outputs get twice as much too. If inputs increase more than or less than twice as much, the returns accordingly increase and decrease, respectively. In many organizations, constant returns to scale assumption are not acceptable. This assumption is appropriate when every institution acts in their optimal level. However, various problems such as competitive effects, constraints, managements' weak performances etc, hinders institutions from acting in optimal scales. Therefore, Banker, Charnes, and Cooper[27] extended BCC version of DEA in 1984, after which varying returns to scale (VRS) are considered in DEA models. This version is known as BCC, taken from the first letters of their names. The mathematical version is as follow:

$$MaxZ_0 = \sum_{r=1}^{s} u_r y_{r0} + w \sum_{i=1}^{m} v_i x_{i0}$$  \hspace{1cm} (30)

$s.t.$

$$\sum_{r=1}^{s} u_r y_{rj} + w \sum_{i=1}^{m} v_i x_{ij} \leq 1$$ \hspace{1cm} \forall j = 1,2,...,n ;$$

$$u_r, v_i \geq 0, w : free \ in \ Sign$$

Equaling the denominator of the objective function with 1, the above non-linear version can be converted to linear model. The $W$, free in sign variable, is the difference between CCR and BCC versions. The $W$ variable in BCC can determine returns to scale for any unit.

If $W < 0$, kind of return to scale is decreasing (Drs).

If $W = 0$, return to scale is constant (Con).

If $W > 0$, return to scale is increasing (Irs).

In Addition to the above VRS model, Banker, Charnes, and Cooper[29] developed
the last version of CCR to include the varying return to scale. Using the constant return to scale will derange the computed amounts for technical efficiency of analysis when all the institutions do not act in optimal scale. Using the varying return to scale leads to very precise analysis, computing technical efficiency based on the amounts of scale efficiency and management efficiency. In the present study, the two preceding models were used to show importance of Fuzzy AHP and DEA integrated effectiveness easily.

3.4. FAHP- DEA Integrated Model

Considering CCR and BCC model, it will be revealed that any $u_r$ and $v_i$, regardless of their inherent values, get considerably different values for different DMUs while their real values are unique for all DMUs in real world. In other words, the DEA technique regardless of its orientation tries to assign maximum or minimum value to $u_r$ and $v_i$, regardless of their real values, to maximize objective function. As clearly it is comprehensible, objective function demonstrates the DMUs' various efficiencies[30]. This means that DEA analysis does not consider the inherent values of inputs and outputs variables, thus the final value of objective function will be overestimated.

Furthermore, regardless of all these influential papers, Yang et al.[31] proved that unrestricted domain of weights rate for output and input variables brings about miss efficiency evaluation through CCR model and then proposed YMK version of DEA model; Although their techniques slightly improved the result of analyses, more that 30% of DMUs took efficiency scores as equal to 1. Anderson and Peterson [32] when saw that the majority of DMUs acquires efficiency rates 1, proposed super efficiency, in which some DMUs could get efficiency rate more that 1 when the target DMU's efficiency fraction was eliminated from constraints. Allen et al.[30] noticed that imposing variable weight restrictions and value judgment considerably improves efficiency rating acceptability by DEA techniques. Although their methods controlled weight fluctuations under believable level, they did not propose efficient algorithm. Bjørndal and Bjørndal [33] in Norwegian electricity distribution companies rating, were forced to use weight restrictions to attain believable consequences and then presented judgmental way for variable weighing. Mansorui et al. [34] used traditional DEA model to prioritize cement industry in Tehran stock exchange and saw that some of the less successful companies got high rate of efficiency scores and concluded that unrestricted weights for input and output variables cause low value variables to get large weights to maximize objective DMU's efficiency falsely. Thus forced to employ TOPSIS(Technique for Order Preference by Similarity to Ideal Solution) model to attain inherent weights. Using TOPSIS model and applying statistical confidence interval constraints, they could control input and output variables weights around their inherent weights. The majority of experts confirmed that the result of TOPSIS-DEA enjoys high validity over traditional DEA model. Thus application of scientific methods for input and output weight fluctuations control will be considerably important.

Because of some qualitative and linguistic criteria, it seems that fuzzy models usage
will be more effective than crisp ones. One of the well-known techniques to rate criteria in fuzzy environment is fuzzy AHP which can effectively cope with criteria and alternative weighing.

To this end, using Fuzzy AHP to determine the inherent weights of inputs and outputs, we developed Fuzzy AHP-DEA integrated method for efficiency evaluation. In the current paper, the CCR and BCC version of DEA is integrated with fuzzy AHP. Fuzzy AHP tries to assess inherent values/weights of inputs and outputs variables. Since the final ranking of criteria will take advantage of sustainability.

The result of Fuzzy AHP was used to attain sample mean, standard deviation and standard error of all input and output variables weights. As central limit theorem teaches, when the number of participant exceeds 30, sample mean will follow a normal distribution and the Eqs.(31) will be established.

\[
P\left(\bar{w}_i - z_{\alpha/2} \times S_{\bar{w}_i} \leq w_i \leq \bar{w}_i + z_{\alpha/2} \times S_{\bar{w}_i}\right) = 1 - \alpha \quad (31)
\]

Where \( \alpha \) is error level, considered 0.05, \( w_i \) shows the mean of criteria \( i \) in the population, \( z_{\alpha/2} \) shows standard normal distribution score, considered 1.96, \( S_{\bar{w}_i} \) is the standard error of weights and \( \bar{w}_i \) is the sample mean, extracted from Eq.(22).

Replacing \( u_r \) and \( v_i \) instead of \( w_i \), Eq.(32) and (33) will be established.

\[
P\left(\bar{u}_r - z_{\alpha/2} \times S_{\bar{u}_r} \leq u_r \leq \bar{u}_r + z_{\alpha/2} \times S_{\bar{u}_r}\right) = 1 - \alpha \quad , \ r = 1,2,..,m; \quad (32)
\]

\[
P\left(\bar{v}_i - z_{\alpha/2} \times S_{\bar{v}_i} \leq v_i \leq \bar{v}_i + z_{\alpha/2} \times S_{\bar{v}_i}\right) = 1 - \alpha \quad , \ i = 1,2,..,n \quad (33)
\]

These two relations explain the range of inherent values of \( u_r \) and \( v_i \) in the population with 95% confidence interval level which means that \( u_r \) and \( v_i \) should not exceed from the specified range. Addition of these constraints to model (30) in the form of standard constraints result in following CCR-output oriented model.

\[
MaxZ_0 = \frac{\sum_{i=1}^{s} u_{r}y_{ij}}{\sum_{j=1}^{m} v_{i}x_{ij}}
\]

\[
\sum_{i=1}^{s} u_{r}y_{ij} \leq 1 \quad \forall j = 1,2,..,n
\]

\[
\sum_{j=1}^{m} v_{i}x_{ij}
\]

\[
\bar{u}_r - z_{\alpha/2} \times S_{\bar{u}_r} \leq u_r \leq \bar{u}_r + z_{\alpha/2} \times S_{\bar{u}_r} \quad , \quad r = 1,2,..,s;
\]

\[
\bar{v}_i - z_{\alpha/2} \times S_{\bar{v}_i} \leq v_i \leq \bar{v}_i + z_{\alpha/2} \times S_{\bar{v}_i} \quad , \quad i = 1,2,..,m
\]

\[
u_{r}, v_{i} \geq 0
\]

Where \( s \) is the number of output variables and \( i \) shows the number of input variables, and incorporating Eqs.(32) and (33) in the form of constraints to model (30), it will change to the following form.
\[
\text{Max} Z_0 = \frac{\sum_{t=1}^{m} u_t y_{r0} + w}{\sum_{i=1}^{n} v_i x_{i0}}
\]

s.t.
\[
\sum_{r=1}^{m} u_r y_{rj} + w \leq 1
\]
\[
\sum_{i=1}^{n} v_i x_{ij}
\]
\[
\bar{u}_r - z\alpha_{1/2} \times S_{\bar{u}_r} \leq u_r \leq \bar{u}_r + z\alpha_{1/2} \times S_{\bar{u}_r}, \quad r = 1, 2, \ldots, s;
\]
\[
\bar{v}_i - z\alpha_{1/2} \times S_{\bar{v}_i} \leq v_i \leq \bar{v}_i + z\alpha_{1/2} \times S_{\bar{v}_i}, \quad i = 1, 2, \ldots, m
\]

\[ j = 1, 2, \ldots, n \]
\[ u_r, v_i \geq 0, w: \text{free in Sign} \]

In these two last models, unknown variables \( u_r \) and \( v_i \) can get values close to their inherent values, determined previously by the expert team. This approach causes Fuzzy AHP-DEA to produce real relative efficiency rate for all DMUs.

To assess differences between technical efficiencies of traditional DEA and fuzzy AHP-DEA integrated model for any DMU based on CCR and BCC version of DEA Eqs.(36) and (37) were defined.
\[
C_i = \text{TETC}_i - \text{TEFC}_i
\]
\[
B_i = \text{TETB}_i - \text{TEFB}_i
\]

Where \( C_i \) shows the differences between technical efficiencies of traditional DEA and Fuzzy AHP-DEA integrated method based on CCR version of DEA for DMU \( i \), \( \text{TETC}_i \) is the technical efficiency of DMU \( i \) based on traditional version of DEA and CCR approach, \( \text{TEFC}_i \) is technical efficiency of DMU \( i \) based on fuzzy AHP-DEA integrated method based on CCR approach, \( B_i \) shows the differences between technical efficiencies of traditional DEA and Fuzzy AHP-DEA integrated method based on BCC version of DEA for DMU \( i \), \( \text{TETB}_i \) is technical efficiency of DMU \( i \) based on traditional version of DEA and BCC approach, \( \text{TEFB}_i \) demonstrates technical efficiency of DMU \( i \) based on fuzzy AHP-DEA integrated method and BCC approach. These differences can be used to determine the homogeneity of changes between two models.

4. Experiment and results

In the current study, we focused on managerial efficiencies evaluation in Iranian I&M bank’s branches. Assessing managerial efficiency improvement needs to determine the main critical success factors. After investigating several related papers, six key ratios were detected. Any changes in these ratios seems not only demonstrates any DMU’s prosperity but also these are directly affected from managerial performance. We introduced these ratios as customers growth, revenue growth, net revenue growth, credit growth, operational cost growth non-operational cost growth and gathered all related data for 2013 calendar year. Furthermore, population density is another factor that affects managerial performance which must be taken into consideration.

The large number for the former four ratios shows managers efficiency improvement while the large amount for the latter two ratios implies managerial deficiency. Also great value of population density makes it difficult to improve managerial performance. In accordance
with DEA literature we defined the former four variable as output variables and the latter three variables as input variables. It is clear that the importance of these variable is not same but DEA tries to assign the best value for all of these variables to maximize any DMU's efficiencies. As mentioned above, this will result in miss efficacy evaluation in many cases. In the present study, inherent weights of these variables were determined by incorporating 30 experts’ opinion through Fuzzy AHP then using confidence interval and central limit theorem, we defined fuzzy AHP DEA integrated model. Thus, experiment phase includes three sections of conducting fuzzy AHP, applying traditional DEA and implementation of Fuzzy AHP- DEA.

4.1. Conducting Fuzzy AHP

To attain inherent priority weights of input and output variables of $u_r$ and $v_i$, fuzzy AHP was used. For this purpose, 30 experts were selected and requested to determine importance of input and output variables' growth in annual performance through Fuzzy AHP's pair wise comparison matrices. For this purpose, pair wise comparisons matrixes are applied to find the inherent weights of input and output variables $-u_r$ and $v_i$ along with performance improvement. Tables (3) and (4) show two of these pair wise comparisons from an expert viewpoint based on Table 1 scales.

Table 3
Sample of Fuzzy AHP pair wise comparisons to determine output variables' inherent weights on performance improvement

<table>
<thead>
<tr>
<th>Performance Improvement</th>
<th>Customers growth</th>
<th>Revenue growth</th>
<th>Net-revenue growth</th>
<th>Credit growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customers growth</td>
<td>(1,1,1)</td>
<td>(3,5,7)</td>
<td>(1,3,5)</td>
<td>(5,7,9)</td>
</tr>
<tr>
<td>Revenue growth</td>
<td>$\left( \frac{1}{7}, \frac{5}{3}, \frac{1}{2} \right)$</td>
<td>(1,1,1)</td>
<td>$\left( \frac{1}{5}, \frac{3}{1}, 1 \right)$</td>
<td>(1,3,5)</td>
</tr>
<tr>
<td>Net revenue growth</td>
<td>$\left( \frac{1}{5}, \frac{3}{1}, 1 \right)$</td>
<td>(1,3,5)</td>
<td>(1,1,1)</td>
<td>(3,5,7)</td>
</tr>
<tr>
<td>Credit growth</td>
<td>$\left( \frac{1}{9}, \frac{7}{5}, \frac{1}{3} \right)$</td>
<td>$\left( \frac{1}{5}, \frac{3}{1}, 1 \right)$</td>
<td>$\left( \frac{1}{7}, \frac{5}{3}, \frac{1}{3} \right)$</td>
<td>(1,1,1)</td>
</tr>
</tbody>
</table>

Table 4
Sample of Fuzzy AHP pair wise comparisons to determine input variables's inherent weights in performance improvement

<table>
<thead>
<tr>
<th>Performance Improvement</th>
<th>Operational cost growth</th>
<th>Non-operational cost growth</th>
<th>Population growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operational cost growth</td>
<td>(1,1,1)</td>
<td>(1,2,4)</td>
<td>$\left( \frac{1}{6}, \frac{4}{2}, \frac{1}{2} \right)$</td>
</tr>
<tr>
<td>Non-operational cost growth</td>
<td>$\left( \frac{1}{4}, \frac{2}{1}, 1 \right)$</td>
<td>(1,1,1)</td>
<td>$\left( \frac{1}{8}, \frac{6}{4}, \frac{1}{2} \right)$</td>
</tr>
</tbody>
</table>
Using Eqs.(16) to (20) and assigning 0.8 to $\alpha$ and 0.1 to $\mu$, all experts' pair wise matrix were converted to the defuzzyfied matrices. Then applying Eq.(21) the priority weight vectors of any expert about the importance of any input and output variables on bank's branches performance were obtained separately. Then applying Eqs.(25) to (27), consistency rate(C.R) for any expert regarding input and output criteria pair wise comparisons matrices were calculated. The result revealed that more than 84% of pair wise matrices enjoy acceptable rate of consistency and for other 16%, the experts were requested to complete their pair wise matrices carefully again. After computing consistency rate of new pair wise comparisons matrices and being ensured about their precision considering their C.R, these matrices were added into the weighing process. After that, using Eqs.(22) to (24) sample mean, standard deviation and standard error of input and output variables weights were computed. Tables (5) and (6) demonstrate all output and input variables' weights for the current study.

<table>
<thead>
<tr>
<th>Output measures</th>
<th>Sample Mean($\bar{u}_r$)</th>
<th>Standard deviation</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customers growth</td>
<td>0.341</td>
<td>0.024</td>
<td>0.004382</td>
</tr>
<tr>
<td>Revenue growth</td>
<td>0.254</td>
<td>0.041</td>
<td>0.007486</td>
</tr>
<tr>
<td>Net revenue growth</td>
<td>0.223</td>
<td>0.035</td>
<td>0.00639</td>
</tr>
<tr>
<td>Credit growth</td>
<td>0.182</td>
<td>0.0142</td>
<td>0.002593</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input measures</th>
<th>Sample Mean($\bar{v}_i$)</th>
<th>Standard deviation</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operational cost growth</td>
<td>0.319</td>
<td>0.035</td>
<td>0.00639</td>
</tr>
<tr>
<td>Non-operational cost growth</td>
<td>0.228</td>
<td>0.065</td>
<td>0.011867</td>
</tr>
<tr>
<td>Population density growth</td>
<td>0.453</td>
<td>0.084</td>
<td>0.015336</td>
</tr>
</tbody>
</table>

4.2. Implementation of traditional data envelopment

After gathering data, we saw that some ratios for several DMUs are negative. Considering that
DEA only applicable with non-negatives variable[35], Eqs.(38) and (39) were used to solve this problem.

\[ Y_{ij} = y_{ij} + \text{ Min } y_{ij} \quad \forall i, j = 1, 2, 3, 4; \]  

\[ X_{ij} = x_{ij} + \text{ Min } x_{ij} \quad \forall i, j = 1, 2; \]

then, replacing new variables instead of primary ratios and applying traditional DEA models of (29) and (30), technical efficiency based on constant return to scale(CRS) and variable return to scale(VRS) for 38 I&M Bank's Branches were computed. However LINGO.14 was applied to help us to analyze integrated DEA-FAHP model because of its considerable ability in programming and in buffering availability. Table 7 shows their various efficiencies.

Table 7
Technical efficiency of Bank's branches using traditional DEA based on CRS and VRS

<table>
<thead>
<tr>
<th>No.</th>
<th>Name of Branches</th>
<th>Technical efficiency based on CRS</th>
<th>Type of return to scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tehran</td>
<td>0.88</td>
<td>Con</td>
</tr>
<tr>
<td>2</td>
<td>Tabriz</td>
<td>0.46</td>
<td>Irs</td>
</tr>
<tr>
<td>3</td>
<td>Sari</td>
<td>0.41</td>
<td>Drs</td>
</tr>
<tr>
<td>4</td>
<td>Zanjan</td>
<td>0.73</td>
<td>Irs</td>
</tr>
<tr>
<td>5</td>
<td>Ardebil</td>
<td>0.65</td>
<td>Drs</td>
</tr>
<tr>
<td>6</td>
<td>Ahvaz</td>
<td>0.51</td>
<td>Irs</td>
</tr>
<tr>
<td>7</td>
<td>Kermansha</td>
<td>1.00</td>
<td>Con</td>
</tr>
<tr>
<td>8</td>
<td>Qom</td>
<td>0.55</td>
<td>Irs</td>
</tr>
<tr>
<td>9</td>
<td>Yazd</td>
<td>0.52</td>
<td>Drs</td>
</tr>
<tr>
<td>10</td>
<td>Bushehr</td>
<td>0.53</td>
<td>Irs</td>
</tr>
<tr>
<td>11</td>
<td>Mashhad</td>
<td>0.61</td>
<td>Drs</td>
</tr>
<tr>
<td>12</td>
<td>Karimkhhan</td>
<td>0.70</td>
<td>Irs</td>
</tr>
<tr>
<td>13</td>
<td>Hafez</td>
<td>1.00</td>
<td>Con</td>
</tr>
<tr>
<td>14</td>
<td>Arak</td>
<td>0.79</td>
<td>Irs</td>
</tr>
<tr>
<td>15</td>
<td>Qazvin</td>
<td>0.86</td>
<td>Irs</td>
</tr>
<tr>
<td>16</td>
<td>Kerman</td>
<td>0.45</td>
<td>Drs</td>
</tr>
<tr>
<td>17</td>
<td>Bandarabas</td>
<td>0.54</td>
<td>Con</td>
</tr>
<tr>
<td>18</td>
<td>Zahedan</td>
<td>0.71</td>
<td>Drs</td>
</tr>
<tr>
<td>19</td>
<td>Semnan</td>
<td>0.63</td>
<td>Irs</td>
</tr>
<tr>
<td>20</td>
<td>Shahrkord</td>
<td>0.65</td>
<td>Drs</td>
</tr>
<tr>
<td>21</td>
<td>Yasuj</td>
<td>0.83</td>
<td>Con</td>
</tr>
<tr>
<td>22</td>
<td>Rasht</td>
<td>0.62</td>
<td>Drs</td>
</tr>
<tr>
<td>23</td>
<td>Gorgan</td>
<td>0.67</td>
<td>Irs</td>
</tr>
<tr>
<td>24</td>
<td>Hamedan</td>
<td>0.56</td>
<td>Drs</td>
</tr>
<tr>
<td>25</td>
<td>Oromiyaeh</td>
<td>0.77</td>
<td>Irs</td>
</tr>
<tr>
<td>26</td>
<td>Khoramaba</td>
<td>0.38</td>
<td>Drs</td>
</tr>
<tr>
<td>27</td>
<td>Sanandaj</td>
<td>0.46</td>
<td>Irs</td>
</tr>
<tr>
<td>28</td>
<td>Eilam</td>
<td>0.43</td>
<td>Drs</td>
</tr>
<tr>
<td>29</td>
<td>Esfahan</td>
<td>1.00</td>
<td>Con</td>
</tr>
<tr>
<td>30</td>
<td>Shiraz</td>
<td>0.66</td>
<td>Irs</td>
</tr>
<tr>
<td>31</td>
<td>Qaem karaj</td>
<td>0.59</td>
<td>Drs</td>
</tr>
<tr>
<td>32</td>
<td>Sanat</td>
<td>0.55</td>
<td>Irs</td>
</tr>
</tbody>
</table>
4.3. Implementation of fuzzy AHP – DEA model

Using models (34) and (35), technical efficiencies of I&M bank's branches were evaluated. Integrated fuzzy AHP- DEA model showed considerable decreases in both types of technical efficiencies. Table 5 demonstrates technical efficiencies of I&M bank's branches using integrated Fuzzy AHP-DEA based on CRS and VRS types of efficiencies using CCR an BCC version of DEA and types of return to scale.

Table 8
Technical efficiencies using integrated fuzzy AHP-DEA model based on CRS and VRS

<table>
<thead>
<tr>
<th>No.</th>
<th>Name of Branches</th>
<th>Technical efficiency based on CRS</th>
<th>Technical efficiency based on VRS</th>
<th>Type of return to scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tehran</td>
<td>0.76</td>
<td>0.81</td>
<td>Drs</td>
</tr>
<tr>
<td>2</td>
<td>Tabriz</td>
<td>0.38</td>
<td>0.49</td>
<td>Irs</td>
</tr>
<tr>
<td>3</td>
<td>Sari</td>
<td>0.38</td>
<td>0.53</td>
<td>Drs</td>
</tr>
<tr>
<td>4</td>
<td>Zanjan</td>
<td>0.54</td>
<td>0.72</td>
<td>Irs</td>
</tr>
<tr>
<td>5</td>
<td>Ardebil</td>
<td>0.56</td>
<td>0.65</td>
<td>Drs</td>
</tr>
<tr>
<td>6</td>
<td>Ahvaz</td>
<td>0.48</td>
<td>0.58</td>
<td>Irs</td>
</tr>
<tr>
<td>7</td>
<td>Kermansha</td>
<td>1.00</td>
<td>1.00</td>
<td>Con</td>
</tr>
<tr>
<td>8</td>
<td>Qom</td>
<td>0.46</td>
<td>0.58</td>
<td>Irs</td>
</tr>
<tr>
<td>9</td>
<td>Yazd</td>
<td>0.47</td>
<td>0.51</td>
<td>Drs</td>
</tr>
<tr>
<td>10</td>
<td>Bushehr</td>
<td>0.51</td>
<td>0.62</td>
<td>Irs</td>
</tr>
<tr>
<td>11</td>
<td>Mashhad</td>
<td>0.57</td>
<td>0.71</td>
<td>Drs</td>
</tr>
<tr>
<td>12</td>
<td>Karimkhan</td>
<td>0.56</td>
<td>0.77</td>
<td>Irs</td>
</tr>
<tr>
<td>13</td>
<td>Hafez</td>
<td>1.00</td>
<td>1.00</td>
<td>Con</td>
</tr>
<tr>
<td>14</td>
<td>Arak</td>
<td>0.79</td>
<td>0.79</td>
<td>Con</td>
</tr>
<tr>
<td>15</td>
<td>Qazvin</td>
<td>0.66</td>
<td>0.68</td>
<td>Irs</td>
</tr>
<tr>
<td>16</td>
<td>Kerman</td>
<td>0.33</td>
<td>0.59</td>
<td>Drs</td>
</tr>
<tr>
<td>17</td>
<td>Bandarabas</td>
<td>0.46</td>
<td>0.51</td>
<td>Irs</td>
</tr>
<tr>
<td>18</td>
<td>Zahedan</td>
<td>0.64</td>
<td>0.72</td>
<td>Drs</td>
</tr>
<tr>
<td>19</td>
<td>Semnan</td>
<td>0.52</td>
<td>0.63</td>
<td>Irs</td>
</tr>
<tr>
<td>20</td>
<td>Shahrdan</td>
<td>0.48</td>
<td>0.67</td>
<td>Drs</td>
</tr>
<tr>
<td>21</td>
<td>Yasuj</td>
<td>0.72</td>
<td>0.81</td>
<td>Drs</td>
</tr>
<tr>
<td>22</td>
<td>Rasht</td>
<td>0.58</td>
<td>0.75</td>
<td>Drs</td>
</tr>
<tr>
<td>23</td>
<td>Gorgan</td>
<td>0.54</td>
<td>0.71</td>
<td>Irs</td>
</tr>
<tr>
<td>24</td>
<td>Hamedan</td>
<td>0.48</td>
<td>0.74</td>
<td>Drs</td>
</tr>
</tbody>
</table>
Sensitivity analysis

Comparing the results of traditional CCR and BCC version of DEA with Fuzzy AHP integrated version shows that there exist significant differences between the findings of two models. For instance, the average of technical efficiency based on constant return to scale from traditional CCR- DEA to CCR- fuzzy AHP-DEA combined model decreased from 0.68 to 0.59. The average of technical efficiency based on variable return to scale from traditional BCC-DEA to BCC-fuzzy AHP-DEA integrated model decreased from 0.75 to 0.67. Fig. 2 compares the technical efficiency of two models based on CCR type of technical efficiency.

Fig. 2
Comparing technical efficiencies based on CRS between traditional DEA and integrated fuzzy AHP-DEA
Fig. 1 shows that technical efficiencies based on CRS using traditional DEA is not less than technical efficiencies based on CRS using Fuzzy AHP Combined model for all DMUs. This means that when $u_r$ and $v_i$ is defined unrestricted, they can get any possible values, regardless of their inherent values, to maximize objective function and to attain maximum efficiency for objective DMU. While in Fuzzy AHP-DEA integrated version, by incorporating inherent values of $u_r$ and $v_i$, extracted from fuzzy AHP, the efficiencies of all DMUs are controlled as a result of applying confidence interval. Thus, the efficiencies of all DMUs approaches to their real efficiencies. Similar results can be seen in BCC version of traditional DEA and fuzzy AHP-DEA integrated models. Fig.3 demonstrates the efficiency differences based on VRS between tradition BCC version of DEA and fuzzy AHP-DEA integrated model.

Comparing technical efficiencies based on VRS using traditional CCR version of DEA and integrated fuzzy AHP-DEA

As seen from the figure above, the technical efficiency based on VRS using traditional BCC
version of DEA is not less than Fuzzy AHP-DEA integrated for all DMUs.

Furthermore, the differences between efficiencies of two model, based on CCR and BCC version considerably affects decision making process. Using Eqs.(36) and (37) the difference between technical efficiencies of two models based on CCR and BCC version were evaluated. Fig. 4 shows the differences between efficiencies of two models based on CCR and BCC version.

Fig. 4
Differences between technical efficiencies of all DMUs using integrated Fuzzy AHP-DEA and traditional DEA based on CCR and BCC version.

As seen from the above figure, the differences between FAHP-DEA integrated and traditional DEA based on CCR and BCC version are significantly different and considerable. For instance, while the difference between two models based on CCR for DMU 37 is equal to 0.1, their difference for the same DMU based on BCC has become 0.23. This means that input and output weight fluctuations’ controller, confidence interval, influence BCC and CCR, differently.

Additionally, return to scale is another important measure that must be taken into consideration. Consistency between two models results in the same decisions but the differences will bring about various actions.

For this purpose we compared the type of return to scale for both models and determined inconsistencies of the models. As demonstrated at the following table, there is only some 15 percent (6 of 38 ) of inconsistency between two models in return to scale evaluation. The comparisons between the results have been depicted at Table 9 and Fig.5.
Table 9
Return to Scale Evaluation by classic DEA and Integrated Model

<table>
<thead>
<tr>
<th>DEA</th>
<th>Constant</th>
<th>Drs</th>
<th>Irs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Drs</td>
<td>0</td>
<td>13</td>
<td>0</td>
</tr>
<tr>
<td>Irs</td>
<td>2</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td>6</td>
<td>16</td>
<td>16</td>
</tr>
</tbody>
</table>

Fig. 5
Comparing return to Scale Evaluation by classic DEA and Integrated Model

As seen in the Fig.1 the result of two models demonstrates approximately same situation from return to scale efficiency point of view.

6. Summary and Conclusion
There are several methods for evaluating efficiency and productivity in organizations. However, the method of data envelopment analysis has been considered as an efficient one. This method envelops all the data and units, determines components of productivity, and computes efficiencies based on many forms. Technical efficiency based on CRS and VRS are dominant form of efficiencies that have been increasingly gained importance. Selecting basic criteria for efficiency analysis is another element, which affects efficiency results. In the current study, the attempt is made to select the most valid variables which are affected by efficiency. Previous studies about efficiency analysis using DEA showed that defining unrestricted ranges for input and output variables result in miss efficiency evaluation so that some low value input/output variables get unreal value to maximize objective DMU’s efficiency. These events in many cases take the efficiency results under question. We used Fuzzy AHP technique to determine inherent weights of input and output variables. Involving 30 experts in Fuzzy AHP analysis process helped us to attain
confidence interval level and central limit theorem. Regarding confidence intervals for $u_r$ and $v_l$ as constraints ensured the author that these variables can not exceed from their real values which population experts believe in. Thus the result of efficiency analyses enjoys the advantage of reality. The new method introduced as fuzzy AHP-DEA integrated method. The result revealed that there exist considerable differences between traditional DEA and fuzzy AHP-DEA integrated considering both CCR and BCC version of DEA model. It is clear that considering inherent weights of $u_r$ and $v_l$, at least in the confidence interval form, guarantees DMUs efficiencies' closeness to their real efficiencies. Also as is shown in the Fig.4, the differences between two models based on CRS and VRS not only were not the same but also considerably significant. Furthermore, incorporating fuzzy AHP and DEA affects types of return to scale too. In this case, similarly, the result of Fuzzy AHP-DEA integrated method brings about different results with traditional DEA, again.

However, computing any type of efficiencies at the branches of I&M bank revealed that the bank branches enjoys moderate to high status related to two types of efficiencies. Thus the I & M Bank’s board of directors must play a prominent role to increase efficiencies. For this purpose, they must adopt strategies to optimize investment and human sources, specialized sources optimally, improve quality of services, satisfy clients, and so on. Finally, the most important result of this paper is to introduce integrated FAHP-DEA method in rankings and efficiency analysis.

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